

A New Model for Strange Star

Debabrata Deb^{a,1}, Sourav Roy Chowdhury^{b,2},
Saibal Ray^{c,3}, Farook Rahaman^{d,4}.

¹Department of Physics, City College, Rammohan Ray Sarani, Kolkata 700009, West Bengal, India

²Department Of Physics, Seth Anandaram Jaipuria College, Raja Naba Krishna Street, Kolkata 900005, West Bengal, India

³Department of Physics, Government College of Engineering & Ceramic Technology, Kolkata 700010, West Bengal, India

⁴Department of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India

Received: date / Accepted: date

Abstract In the present letter we attempt to find a new class of solutions relating radiation model by using Homotopy Perturbation Method (HPM). We consider that the spherical distribution is made of perfect fluid where radiation is along the radially outward direction of the system. Here we have employed HPM as a new tool for astrophysical systems which via the mass polynomial facilitates solving of field equations. A set of interior solutions has been found out on the basis of the equation of state in the form $p = \frac{1}{3}\rho$ with the demand that the radiation model is consistent with the Schwarzschild metric. It is observed that the set of solutions thus developed provides new metric to explain strange star which however seems not free from singularity.

Keywords general relativity; Homotopy Perturbation Method; compact stars

1 Introduction

Under the general relativistic astrophysics the composition, nature and physical properties of compact stars have recently become a more fascinating research arena with diverse theoretical as well as applied issues. Several scientists [1,2,3] have been pointed out that the quark matter, made of u, d and s quarks, which is known as the strange quark matter may be more stable than ordinary nuclear matter. This strange matter hypothesis actually indicates important properties of highly compact objects like neutron star and dwarf star. Essentially strange (quark) matter is a kind of Quark-Gluon-Plasma (QGP) which may be composed of equal numbers of u, d and s quarks along with a small number of electrons and some baryons. The strange stars (SS) named after the strange quark are therefore composed of strange matter.

Using the MIT bag model Farook et al. [4] have obtained a deterministic model of strange star where they considered a mass polynomial and analyzed all physical

^ae-mail: d.deb32@gmail.com

^be-mail: b.itsme88@gmail.com

^ce-mail: saibal@iucaa.ernet.in

^de-mail: rahaman@iucaa.ernet.in

properties of strange star. However they were unable to show any properties up to 6 km from the centre of the system.

In this present letter we have constructed a model to discuss the physical properties of strange star altogether from the centre to the surface of the star. Solving the non-linear equations analytically has always been a challenge in astrophysics. The Homotopy Perturbation Method (HPM) is a powerful and very simple tool to solve these kind of equations with least number of assumptions. In the present work modeling the strange star we have developed expression of mass (mass polynomial) which is radial function of r . However, it is not just assumed arbitrarily rather we have done this by the help of HPM. Later on we have substituted that expression of mass to solve Einstein's Field Equations.

Therefore, following Rahaman et al. [5] we have obtained the interior solution of the Einstein field equation of a spherically symmetric system of radiating star by introducing a little bit correction. In that paper [5] it was concluded that the model can not be compatible with the compact stars. In the present work however, with some appropriate changes, we have been able to justify the physical properties of the compact object, especially strange stars. We also would like to refer that the work of Rahaman et al. [5] where Homotopy Perturbation Method has been employed for a spherically symmetric system of radiating star suffers from instability problem. In the present letter our motivation is to present a stable model of a radiating compact star.

The outline of our investigation is as follows: In the Sec. 2 we have calculated mass of the system using firstly, the Maximum Entropy Principle, and secondly, the Homotopy Perturbation Method. Sec. 3 deals with the solution of Einstein's field equations for different physical parameters, viz. the pressures (p_r and p_t) and density. We have discussed and explored several physical features in Sec. 4 and a comparative study has been conducted in Sec. 5 for validity of the data set of the present model with that of the exiting strange stars available in the literature. In the last sec. 6 we have drawn some concluding remarks with some salient features of the present model.

2 Calculation of mass of the spherical system

2.1 The Maximum Entropy Principle

The thermal radiation of a perfect fluid sphere satisfies following the equation of state (EOS)

$$p_r = \frac{1}{3}\rho. \quad (1)$$

It is to note that since the fluid is anisotropic so p_t and p_r are different but we have used Eq. (1) as our concern is the radiation model.

The other necessary equations for the perfect fluid system are as follows:

$$s = \frac{4}{3}bT^3, \quad (2)$$

$$\rho = \frac{4}{3}bT^4. \quad (3)$$

Here s is the entropy density of the system, T is the temperature of the black body radiation, ρ is the matter density of the system and b is an arbitrary constant of order unity (in natural units $G = c = h = k = 1$).

We consider the interior space-time metric of the spherical symmetric system as

$$ds^2 = -g_{tt}(r)dt^2 + \left[1 - \frac{2m(r)}{r}\right]^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (4)$$

where $m(r)$ is the mass distribution of the system. The time-time component of the metric g_{tt} is the function of radial component r only.

The general energy-momentum tensor for the spherically symmetric radiating perfect fluid system is as follows

$$T_{\nu}^{\mu} = (\rho + \frac{1}{3}\rho)u^{\mu}u_{\nu} - \frac{1}{3}\rho g_{\nu}^{\mu} + (p_t - \frac{1}{3}\rho)\eta^{\mu}\eta_{\nu}, \quad (5)$$

with $u^{\mu}u_{\nu} = -\eta^{\mu}\eta_{\nu} = 1$.

Here the vector u^{μ} is the fluid 4-velocity of the local rest frame and η^{μ} is the unit space-like vector of the radiating system, and the vectors are orthogonal to each other i.e. $u^{\mu}\eta_{\nu} = 0$.

The constraint relation comes from the time component of Einstein field equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$ is

$$\rho = \frac{m'(r)}{4\pi r^2}. \quad (6)$$

From Eq. (2), Eq. (3) and Eq. (6) we have

$$s = \frac{4}{3}b^{\frac{1}{4}}\left(\frac{m'(r)}{4\pi r^2}\right)^{\frac{3}{4}}. \quad (7)$$

Now one can write the total entropy of the spherical system for the matter distribution up to $r \leq R$ as

$$S = \int_V s(r) \left(1 - \frac{2m(r)}{r}\right)^{-1/2} dV = \alpha \int_0^R L dr, \quad (8)$$

where $\alpha = \frac{4}{3}4\pi b^{\frac{1}{4}}$ and the Lagrangian of the system as

$$L = (m')^{\frac{3}{4}}r^{\frac{1}{2}}\left(1 - \frac{2m(r)}{r}\right)^{-\frac{1}{2}}. \quad (9)$$

From the Euler-Lagrangian equation of motion

$$\frac{\partial}{\partial r} \left(\frac{\partial L}{\partial m'} \right) - \frac{\partial L}{\partial m} = 0, \quad (10)$$

we therefore obtain

$$-\frac{3}{16}m''r + \frac{3}{8}m''m + \frac{3}{8}m' - \frac{3}{2}\frac{m'm}{r} - \frac{1}{4}(m')^2 = 0. \quad (11)$$

The above Lagrangian equation of motion is a non-linear differential equation. By solving this equation one would get the structure of the self gravitating radiation system.

2.2 The Homotopy Perturbation Method

We have calculated the value of m by using *Homotopy Perturbation Method (HPM)* as provided by He [6].

The Homotopy for any non-linear differential equation is

$$L(m) - L(m_0) + p[L(m_0) + N(m)] = 0, \quad (12)$$

where p is a parameter, such that $p = 0$ gives the initial approximation solution and $p = 1$ gives the desired solution for the mass. The term $L(m) = \frac{dm}{dr} = m'$ is considered as the linear part of the equation and rest of the terms are non-linear. Also, it is adopted that $L(m_0) = m'_0$. We choose initial solution for this system as $m_0 = ar^3$, where a is a numerical constant. As a boundary condition we have assumed that the density should be minimum at the surface of the compact star so that $d(\rho)/dr = 0$ at the surface. One can see later on that after plotting different values of a (among three different values of a for a specific set of data), for a specific value of a only, the density becomes minimum at the surface.

To find out the value of m , we consider the general solution of m as

$$m = m_0 + p^1 m_1 + p^2 m_2 + \dots \quad (13)$$

Now from the Eq. (11), Eq.(12) and Eq. (13) we obtain

$$\begin{aligned} m &= \lim_{p \rightarrow 1} (m_0 + p^1 m_1 + p^2 m_2 + \dots) \\ &= ar^3 + \frac{36}{5} a^2 r^5 + \frac{312}{35} a^3 r^7 + 3ac_1 r^2 + c_2 \end{aligned} \quad (14)$$

where c_1 and c_2 are the integrating constants which will be evaluated later on in the next Sec. 3.

3 The Solution of Einstein's field equations

The Einstein Field Equation for the metric (4), for the matter distribution given in Eq. (1) can be written as

$$\frac{2m'}{r^2} = 8\pi\rho, \quad (15)$$

$$\frac{2m}{r^3} - \left(1 - \frac{2m}{r}\right) \frac{g'_{tt}}{g_{tt}} \frac{1}{r} = -\frac{8\pi}{3}\rho, \quad (16)$$

$$\begin{aligned} & - \left(1 - \frac{2m}{r}\right) \left[\frac{1}{2} \frac{g''_{tt}}{g_{tt}} - \frac{1}{4} \left(\frac{g'_{tt}}{g_{tt}}\right)^2 + \frac{1}{2r} \frac{g'_{tt}}{g_{tt}} \right] \\ & - \left(\frac{m}{r^2} - \frac{m'}{r}\right) \left(\frac{1}{r} + \frac{1}{2} \frac{g'_{tt}}{g_{tt}}\right) = -8\pi p_t. \end{aligned} \quad (17)$$

From Eq. (15), after substitution of Eq. (14), we get the density of the system as

$$\rho = \frac{m'}{4\pi r^2} = \frac{1}{4\pi} \left[3a + 36a^2 r^2 + \frac{312}{5} a^3 r^4 + \frac{6ac_1}{r} \right]. \quad (18)$$

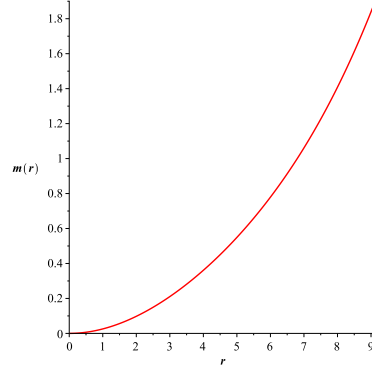


Fig. 1 Variation of mass as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$

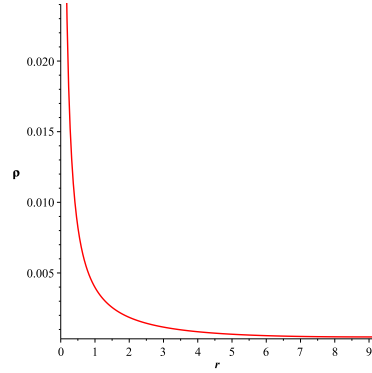


Fig. 2 Variation of density as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$

Let us now choose $\rho' = 0$ at the surface ($r = R$) of the star, i.e. the density is minimum at the surface. By imposing this boundary condition in Eq. (18), we obtain

$$c_1 = 12aR^3 + \frac{208}{5}a^2R^5, \quad (19)$$

and from the boundary condition $m = 0$ at the centre $r = 0$, we get

$$c_2 = 0. \quad (20)$$

Therefore the mass of the spherical system, due to Eqs. (14), (19) and (20), becomes

$$m = ar^3 + \frac{36}{5}a^2r^5 + \frac{312}{35}a^3r^7 + 3ar^2(12aR^3 + \frac{208}{5}a^2R^5). \quad (21)$$

Hence Eq. (18), by the help of (19), takes the form

$$\rho = \frac{1}{4\pi} \left[3a + 36a^2r^2 + \frac{312}{5}a^3r^4 + \frac{6a}{r}(12aR^3 + \frac{208}{5}a^2R^5) \right]. \quad (22)$$

The behaviour of the mass and density are shown in Figs. 1 and 2.

Now from the EOS (1) and Eq. (22) one get

$$p_r = \frac{1}{4\pi} \left[a + 12a^2r^2 + \frac{104}{5}a^3r^4 + \frac{2a}{r}(12aR^3 + \frac{208}{5}a^2R^5) \right], \quad (23)$$

$$g_{tt} = C \frac{e^{\nu(r)}}{\left(1 - \frac{2m(r)}{r}\right)^{\frac{1}{3}}}, \quad (24)$$

where $\nu(r) = \frac{4}{3} \int \frac{\frac{2m}{r^2}}{\left(1 - \frac{2m}{r}\right)} dr$. After evaluating C , based on suitable boundary condition, Eq. (24) can be written as

$$g_{tt} = e^{[\nu(r) - \nu(R)]} \frac{\left(1 - \frac{2M}{R}\right)^{\frac{4}{3}}}{\left(1 - \frac{2m(r)}{r}\right)^{\frac{1}{3}}}. \quad (25)$$

This is the interior time-time component of the metric tensor for the radiating spherical system.

Now the tangential component of the pressure can be obtained as

$$p_t = -\frac{1}{32\pi r^3 g_{tt}^2} (4g_{tt}^2 m_r r - 4g_{tt}^2 m - 2g_{tt}' g_{tt} r^2 + 2r g_{tt}' g_{tt} m + 2g_{tt}' g_{tt} r^2 m_r - 2g_{tt}'' g_{tt} r^3 + 4g_{tt}'' g_{tt} r^2 m + g_{tt}''^2 r^3 - 2g_{tt}'^2 r^2 m). \quad (26)$$

After substitution of the values of g_{tt} , g_{tt}' and g_{tt}'' , the above equation becomes

$$p_t = \frac{-12mm_r + 6mr m_{r,r} - 4rm_r^2 - 3r^2 m_{r,r}}{72r^2(-r + 2m)\pi} \quad (27)$$

where m_r , $m_{r,r}$ are the 1st and 2nd order derivative of the mass with respect to r .

4 Physical properties of the radiation model

4.1 Anisotropy of the system

We have already found out the expressions for the radial as well as tangential pressures in Eqs. (23) and (27). In terms of these pressures therefore the anisotropy of the system can be measured as

$$\Delta = p_t - p_r = \frac{-12mm_r + 6mr m_{r,r} - 4rm_r^2 - 3r^2 m_{r,r}}{72r^2(-r + 2m)\pi} - \frac{m_r}{12\pi r^2}. \quad (28)$$

The nature of the pressures and anisotropy are shown in Fig. 3. which in general show the physically acceptable features.

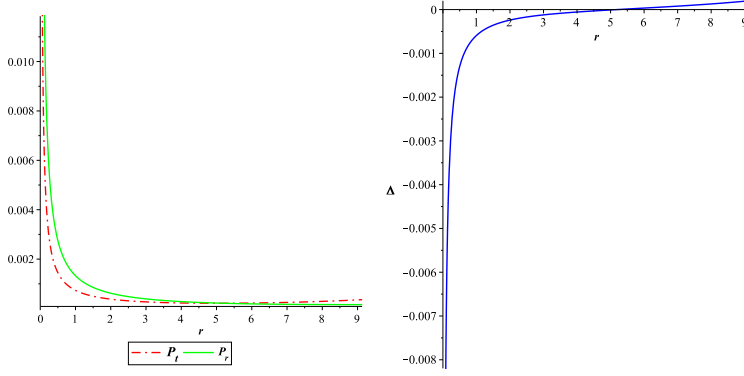


Fig. 3 Variation of radial and transverse pressures as well as the anisotropy as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$

4.2 Stability of the system

4.2.1 The Tolman-Oppenheimer-Volkoff equation

To study the stability of the system we have checked the stability equation given by Tolman [7], Oppenheimer and Volkoff [8]. The TOV equation depicts the equilibrium condition of a star subject to the gravitational force, hydrostatic force and anisotropic force. The generalized TOV equation can be written as [9,10]

$$-\frac{M_g(\rho + p_r)}{r^2} e^{\frac{\lambda-\gamma}{2}} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0, \quad (29)$$

where the effective gravitational mass M_g of the system is defined as

$$M_g = \frac{1}{2} r^2 e^{\frac{\gamma-\lambda}{2}} \gamma'. \quad (30)$$

The TOV equation for our system can be translated as

$$-\frac{2}{3} \rho \frac{g'_{tt}}{g_{tt}} - \frac{1}{3} \frac{d\rho}{dr} + \frac{2}{r}(p_t - p_r) = 0, \quad (31)$$

where the 1st term of the above equation is gravitational force, the 2nd term is the hydrostatic force and the last term is the anisotropic force respectively, so that for equilibrium of the system we should have

$$F_g + F_h + F_a = 0. \quad (32)$$

We have drawn the forces in Fig. 4 which describes the overall action of the three different forces and how the equilibrium is achieved.

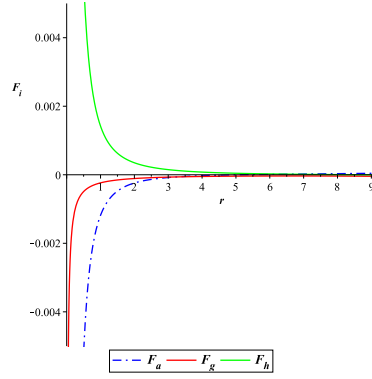


Fig. 4 Variation of gravitational force, anisotropic force and hydrostatic force as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$.

4.2.2 The status of the sound velocity within the system

To examine the stability of the system here we have used the cracking technique [11]. The condition of causality gives the physically accepted conditions for fluid distribution. It states that square of the tangential and radial velocities must lie within the limit 0 and 1. The Herrera's cracking concept states that for a stable region there the radial sound speed must be greater than the transverse sound speed and also in the interior of the matter distribution two sound speed difference i.e. $v_{st}^2 - v_{sr}^2$ should maintain its sign same everywhere i.e 'no cracking'. In our work for the specified sets of data we find that $0 \leq v_{st}^2 \leq 1$ and $0 \leq v_{sr}^2 \leq 1$ (see Top Left panel of Fig. 5). As $|v_{st}^2 - v_{sr}^2| \leq 1$ hence our system is stable according to the works of Herrera [11] and Andréasson [12]. So from Fig. 5 (rest of the two panels) it is clear that our model is perfectly stable.

4.3 Energy conditions

The radiating spherically symmetric system should satisfy all the energy conditions, viz. null energy condition (NEC), weak energy condition (WEC) and strong energy condition (SEC) respectively given by

$$(i) \text{ NEC : } \rho + p_r \geq 0, \rho + p_t \geq 0,$$

$$(ii) \text{ WEC : } \rho + p_r \geq 0, \rho \geq 0, \rho + p_t \geq 0,$$

$$(iii) \text{ SEC : } \rho + p_r \geq 0, \rho + p_r + 2p_t \geq 0.$$

In Fig. 6 we have shown the behaviour of all the above mentioned energy inequalities.

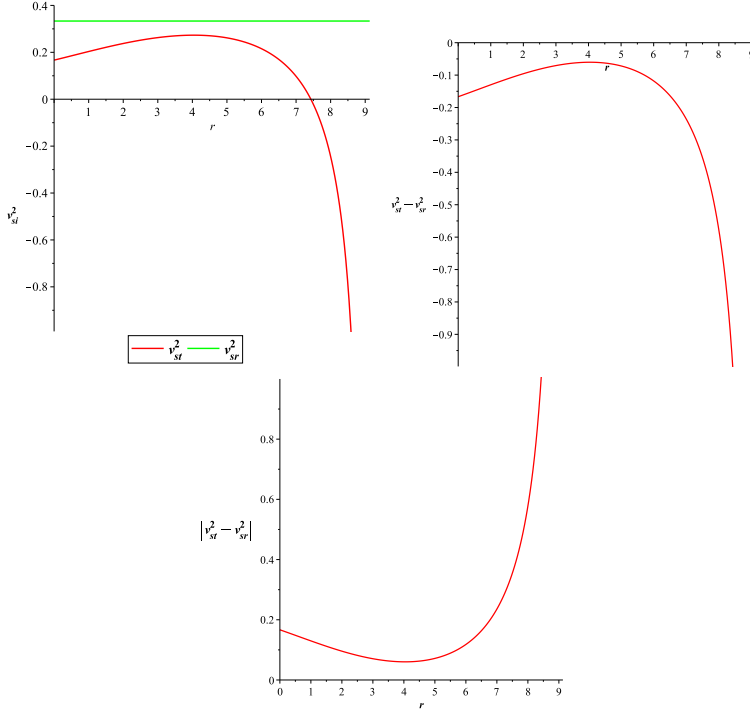


Fig. 5 Variation of sound speeds as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$

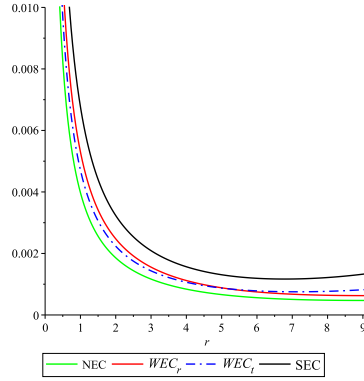


Fig. 6 Variation of different energies as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$

4.4 Surface Redshift

The compactness of a star is defined as the mass-to-radius ratio of the system, i.e. $u(r) = m(r)/r$. And according to the condition of Buchdahl [13] the maximum allowed mass radius ratio is $\leq 4/9$ (≈ 0.4444) for the perfect fluid sphere.

For our system the compactness is

$$u(r) = \frac{1}{35}ar(35r + 252ar^3 + 312a^2r^5 + 1260aR^3 + 4368a^2R^5). \quad (33)$$

Surface red shift(Z) of a star is defined as

$$1 + Z = [1 - 2u(r)]^{-\frac{1}{2}}, \quad (34)$$

which is for the above studied system as follow

$$Z = \frac{1}{\sqrt{1 - (2ar^2 + \frac{72}{5}a^2r^4 + \frac{624}{35}a^3r^6 + 72a^2rR^3 + \frac{1248}{5}a^3rR^5)}} - 1. \quad (35)$$

The compactness and surface redshift are shown in Fig. 7. From the X-ray spectrum the surface redshift Z can be easily observed and correspondingly compactness can be calculated.

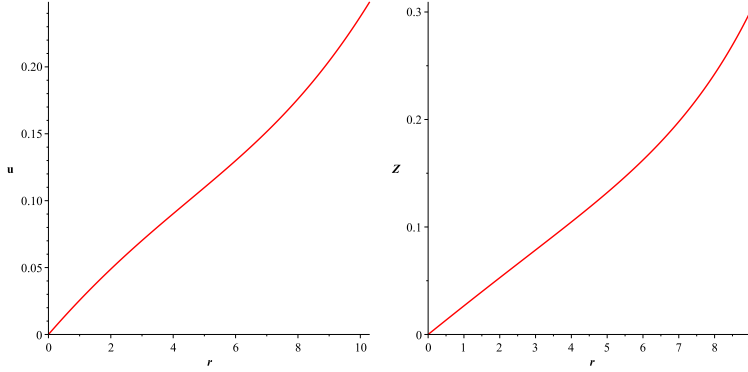


Fig. 7 Variation of compactness and redshift as a function of radial distance r for $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$

5 A comparative study

To study the physical properties of the system we choose the parameter $a = -0.1233 \times 10^{-2} \text{ km}^{-2}$ and $R = 9.13 \text{ km}$ and mass $m(R) = 1.29$ solar mass (SMC X-1). We choose this type of data set as mentioned by Farook et al. [4] because our target is to investigate a radiating model which is compatible with compact stars.

Using the above numerical values of different properties of the interior solution of spherical symmetric radiating body we have calculated each of them and also the energy densities of the strange star. From Fig. 6 it is clear that our model satisfy all the energy conditions and also other physical parametric conditions. We consider here a predicted star of radius 9.9 km and mass 1.71 solar mass. The surface density of the star is obtained as $\rho = 6.73 \times 10^{14} \text{ gcm}^{-3}$ which confirms validity of the model with the observational evidence [14, 15, 16]. However, our model

Table 1 Physical parameters

Strange Stars	Radius (in km)	Mass (M_{\odot})	Mass (in km)
PSR J1614-2230	10.3	1.97	2.9057
Vela X - 12	9.99	1.77	2.6107
PSR J1903+327	9.82	1.667	2.4588
Cen X - 3	9.51	1.49	2.1977
SMC X - 1	9.13	1.29	1.9027

cannot predict the central density as at the center there occurs singularity. The compactness of the predicted star is 0.2553 and corresponding redshift is 0.4295. This high redshift is quite relevant for strange stars. Obviously our predicted star have more compactness than neutron stars. Cackett et al. [17] have reported that low mass X-ray binary 4U1820 – 30 has gravitational redshift as $Z = 0.43$. This indicates that the model studied here is a representative of a compact star and is suitable to explore different properties of strange stars.

6 Conclusion

In this paper we have tried to find out an expression of mass distribution of spherical symmetric radiating system by using HPM of Euler-Lagrangian equation. We have obtained an expression of interior solution of spherically symmetric radiating body and studied different physical properties of the system.

The present investigation reveals that -

(1) Our radiating model is compatible with the compact stars, specially that of strange stars as can be seen from the Comparative Study part of the previous Sec. 5.

(2) From Fig. 3 we observe that the radial and tangential pressures are decreasing with radial distance. However, up to certain distance from the center of the star the radial pressure becomes greater than the transverse pressure. Thus anisotropy is negative up to that region and after that it is increasing with radial distance up to the surface. This particular aspect of the last panel of Fig. 3 actually shows that after 5 km, its sign changes i.e. $\Delta < 0$ to $\Delta > 0$ which implies that the anisotropic force changes from attractive to repulsive. This may be explored physically as follows that the fluid sphere taking its shape from oblate spheroid to prolate spheroid. In this connection we note reports on the non-spherical models of compact stars, especially neutron stars [18,19,20,21,22,23]. We also note the recent work [24] which shows that high magnetic fields in proto-quark stars modify quark star masses. As we have modeled here features of the compact stars composed of strange quark matters so we suspect similar non-spherical effect may be arising in the present study also. However, it needs further investigation.

(3) Although our model satisfies almost all the physical properties and shows stability of the system however seems not free from singularity. The probable origin of singularity may be hidden within the assumption we used in the study. In the total entropy of the system S of Eq. (8) we choose the radial component in the form $(1 - 2m/r)^{-1/2}$, the time component being unknown in the interior region of the sphere. This has been done purposely otherwise number of unknowns exceed

the number of equations which are not at all solvable. Since such form of the metric is intrinsically singular so there is a possibility our metric to become infected too.

Acknowledgments

SR and FR are thankful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune, India for providing Visiting Associateship under which a part of this work was carried out.

References

1. A.R. Bodmer, Phys. Rev. D **4**, 1601 (1971).
2. H. Terazawa, INS Report 336, Tokio University (1979).
3. E. Witten, Phys. Rev. D **30**, 272 (1984).
4. F. Rahaman, K. Chakraborty, P.K.F. Kuhfittig, G.C. Shit, M. Rahman, Eur. Phys. J. C **74**, 3126 (2014).
5. F. Rahaman, S. Ray, A. Aziz, S. Roy Chowdhury, D. Deb, arXiv: 1504.05838 (2015).
6. J.H. He, Int. J. Nonlinear Mech. **35**, 115 (2000).
7. R.C. Tolman, Phys. Rev. **55**, 364 (1939).
8. J.R. Oppenheimer, G.M. Volkoff, Phys. Rev. **55**, 374 (1939).
9. J. Ponce de Leon, Gen. Relativ. Gravit. **25**, 1123 (1993).
10. V. Varela, F. Rahaman, S. Ray, K. Chakraborty, M. Kalam, Phys. Rev. D **82**, 044052 (2010).
11. L. Herrera, Phys. Lett. A **165**, 206 (1992).
12. H. Andréasson, Commun. Math. Phys. **288**, 715 (2009).
13. H.A. Buchdahl, Phys. Rev. **116**, 1027 (1959).
14. R. Ruderman, Rev. Astr. Astrophys. **10**, 427 (1972).
15. N.K. Glendenning, Compact Stars: Nuclear Physics, Particle Physics and General Relativity, Springer-Verlag, New York, p. 70 (1997).
16. M. Herjog, F.K. Roepke, arXiv: 1109.0539 [astro-ph.HE] (2014).
17. E.M. Cackett et al., ApJ **674**, 415 (2008).
18. S. Chandrasekhar, E. Fermi, ApJ **118**, 116 (1953).
19. V.C.A. Ferraro, ApJ **119**, 407 (1954).
20. M. Goosens, Astrophys. Space Sci. **16**, 286 (1972).
21. J.I. Katz, MNRAS **239**, 751 (1989).
22. D.J.B. Payne, A. Melatos, MNRAS **351**, 569 (2004).
23. B. Haskell, S. Samuelsson, K. Glampedakis, N. Andersson, MNRAS **385**, 531 (2008).
24. V. Dexheimer, D.P. Menezes, M. Strickland, J. Phys. G: Nucl. Part. Phys. **41**, (2014) 015203.

